Czy to jest możliwe

Wyznacz dokładne wartości następujących funkcji trygonometrycznych: sin6°; cos6°; tan6°; sin18°; cos18°; tan18°; sin39°; cos39°; tan39°; sin51°; cos51°; tan51°

Rozwiązanie:

$$\sin 6° = \sin(36° - 30°) = \sin 36° \cos 30° - \cos 36° \sin 30° =$$

$$\sin 36° = \sqrt{1-\cos^2 36°} = \sqrt{1-\left(\frac{\sqrt{5}+1}{4}\right)^2} = \sqrt{1-\frac{5+2\sqrt{5}+1}{16}} = \sqrt{1-\frac{6+2\sqrt{5}}{16}} =$$

$$= \sqrt{1-\frac{3+\sqrt{5}}{8}} = \sqrt{\frac{8-3-\sqrt{5}}{8}} = \sqrt{\frac{5-\sqrt{5}}{8}} = \frac{1}{2}\sqrt{\frac{5-\sqrt{5}}{2}}$$

$$\sin 36° \cos 30° - \cos 36° \sin 30° = \frac{1}{2}\sqrt{\frac{5-\sqrt{5}}{2}} \cdot \frac{\sqrt{3}}{2} - \frac{\sqrt{5}+1}{4} \cdot \frac{1}{2} = \frac{\sqrt{3}}{4}\sqrt{\frac{5-\sqrt{5}}{2}} - \frac{\sqrt{5}+1}{8}$$

$$\cos 6° = \cos(36° - 30°) = \cos 36° \cos 30° + \sin 36° \sin 30° =$$

$$= \frac{\sqrt{5}+1}{4} \cdot \frac{\sqrt{3}}{2} + \frac{1}{2}\sqrt{\frac{5-\sqrt{5}}{2}} \cdot \frac{1}{2} = \frac{\sqrt{15}+\sqrt{3}}{8} + \frac{1}{4}\sqrt{\frac{5-\sqrt{5}}{2}}$$

$$\tan 6° = \frac{\sin 6°}{\cos 6°} = \frac{\frac{\sqrt{3}}{4}\sqrt{\frac{5-\sqrt{5}}{2}} - \frac{\sqrt{5}+1}{8}}{\frac{\sqrt{15}+\sqrt{3}}{8} + \frac{1}{4}\sqrt{\frac{5-\sqrt{5}}{2}}} = \frac{2\sqrt{3} \cdot \sqrt{\frac{5-\sqrt{5}}{2}} - \sqrt{5} - 1}{\sqrt{15}+\sqrt{3}+2 \cdot \sqrt{\frac{5-\sqrt{5}}{2}}}$$

$$\sin 18° = \sin(90° - 72°) = \sin 90° \cos 72° - \cos 90° \sin 72° = \cos 72° = \frac{\sqrt{5}-1}{4}$$

$$\cos 18° = \cos(90° - 72°) = \cos 90° \cos 72° + \sin 90° \sin 72° = \sin 72°$$

$$\sin 72° = \sqrt{1-\cos^2 72°} = \sqrt{1-\left(\frac{\sqrt{5}-1}{4}\right)^2} = \sqrt{1-\frac{5-2\sqrt{5}+1}{16}} = \sqrt{1-\frac{6-2\sqrt{5}}{16}} =$$

$$= \sqrt{1-\frac{3-\sqrt{5}}{8}} = \sqrt{\frac{8-3+\sqrt{5}}{8}} = \sqrt{\frac{5+\sqrt{5}}{8}} = \frac{1}{2} \cdot \sqrt{\frac{5+\sqrt{5}}{2}}$$

$$\tan 18° = \frac{\sin 18°}{\cos 18°} = \frac{\frac{\sqrt{5}-1}{4}}{\frac{1}{2} \cdot \sqrt{\frac{5+\sqrt{5}}{2}}} = \frac{\frac{\sqrt{5}-1}{2}}{\sqrt{\frac{5+\sqrt{5}}{2}}}$$

$$\sin 39° = \sin(36° + 6°) = \sin 36° \cos 6° + \cos 36° \sin 6° =$$

$$=\frac{1}{2}\sqrt{\frac{5-\sqrt{5}}{2}}\cdot\left(\frac{\sqrt{15}+\sqrt{3}}{8}+\frac{1}{4}\sqrt{\frac{5-\sqrt{5}}{2}}\right)+\frac{\sqrt{5}+1}{4}\cdot\left(\frac{\sqrt{3}}{4}\sqrt{\frac{5-\sqrt{5}}{2}}-\frac{\sqrt{5}+1}{8}\right)$$

$$\cos 39° = \cos(36° + 6°) = \cos 36° \cos 6° - \sin 36° \sin 6° =$$

$$=\frac{\sqrt{5}+1}{4}\cdot\left(\frac{\sqrt{15}+\sqrt{3}}{8}+\frac{1}{4}\sqrt{\frac{5-\sqrt{5}}{2}}\right)-\frac{1}{2}\sqrt{\frac{5-\sqrt{5}}{2}}\cdot\left(\frac{\sqrt{3}}{4}\sqrt{\frac{5-\sqrt{5}}{2}}-\frac{\sqrt{5}+1}{8}\right)$$

$$\tan 39° = \frac{\frac{1}{2}\sqrt{\frac{5-\sqrt{5}}{2}}\cdot\left(\frac{\sqrt{15}+\sqrt{3}}{8}+\frac{1}{4}\sqrt{\frac{5-\sqrt{5}}{2}}\right)+\frac{\sqrt{5}+1}{4}\cdot\left(\frac{\sqrt{3}}{4}\sqrt{\frac{5-\sqrt{5}}{2}}-\frac{\sqrt{5}+1}{8}\right)}{\frac{\sqrt{5}+1}{4}\cdot\left(\frac{\sqrt{15}+\sqrt{3}}{8}+\frac{1}{4}\sqrt{\frac{5-\sqrt{5}}{2}}\right)-\frac{1}{2}\sqrt{\frac{5-\sqrt{5}}{2}}\cdot\left(\frac{\sqrt{3}}{4}\sqrt{\frac{5-\sqrt{5}}{2}}-\frac{\sqrt{5}+1}{8}\right)}$$

$$\sin 51° = \cos 39° =$$

$$=\frac{\sqrt{5}+1}{4}\cdot\left(\frac{\sqrt{15}+\sqrt{3}}{8}+\frac{1}{4}\sqrt{\frac{5-\sqrt{5}}{2}}\right)-\frac{1}{2}\sqrt{\frac{5-\sqrt{5}}{2}}\cdot\left(\frac{\sqrt{3}}{4}\sqrt{\frac{5-\sqrt{5}}{2}}-\frac{\sqrt{5}+1}{8}\right)$$

$$\cos 51° = \sin 39° =$$

$$\frac{1}{2}\sqrt{\frac{5-\sqrt{5}}{2}}\cdot\left(\frac{\sqrt{15}+\sqrt{3}}{8}+\frac{1}{4}\sqrt{\frac{5-\sqrt{5}}{2}}\right)+\frac{\sqrt{5}+1}{4}\cdot\left(\frac{\sqrt{3}}{4}\sqrt{\frac{5-\sqrt{5}}{2}}-\frac{\sqrt{5}+1}{8}\right)$$

$$\tan 51° = \frac{\frac{\sqrt{5}+1}{4}\cdot\left(\frac{\sqrt{15}+\sqrt{3}}{8}+\frac{1}{4}\sqrt{\frac{5-\sqrt{5}}{2}}\right)-\frac{1}{2}\sqrt{\frac{5-\sqrt{5}}{2}}\cdot\left(\frac{\sqrt{3}}{4}\sqrt{\frac{5-\sqrt{5}}{2}}-\frac{\sqrt{5}+1}{8}\right)}{\frac{1}{2}\sqrt{\frac{5-\sqrt{5}}{2}}\cdot\left(\frac{\sqrt{15}+\sqrt{3}}{8}+\frac{1}{4}\sqrt{\frac{5-\sqrt{5}}{2}}\right)+\frac{\sqrt{5}+1}{4}\cdot\left(\frac{\sqrt{3}}{4}\sqrt{\frac{5-\sqrt{5}}{2}}-\frac{\sqrt{5}+1}{8}\right)}$$